Section 3.4 Properties of Logarithms

Change-Base Formula:

Let *a*, *b*, and *x* be positive real numbers such that a ≠ 1 and b ≠ 1. Then $\log_a x$ can be converted to a different base as follows.

Base *b*	**Base 10**	**Base *e***
$\log_a x = \frac{\log_b x}{\log_b a}$	$\log_a x = \frac{\log x}{\log a}$	$\log_a x = \frac{\ln x}{\ln a}$

Ex 1: Evaluate each of the following, using the change-of-base formula with common logs. Approximate to four decimal places.

a. $\log_3 16$ b. $\log_5 22$

Ex 2: Evaluate each of the following, using the change-of-base formula with natural logarithms. Approximate to four decimal places.

a. $\log_3 16$ b. $\log_5 22$

Properties of Logarithms

Let a be a positive number such that $a \neq 1$, and let n be a real number. If u and v are positive real numbers, then the following properties are true.

	Logarithm with Base *a*	**Natural Logarithm**
1. Product Property:	$\log_a(uv) = \log_a u + \log_a v$	$\ln(uv) = \ln u + \ln v$
2. Quotient Property:	$\log_a \frac{u}{v} = \log_a u - \log_a v$	$\ln \frac{u}{v} = \ln u - \ln v$
3. Power Property:	$\log_a u^n = n \log_a u$	$\ln u^n = n \ln u$

Ex 3: Write each logarithm in terms of ln2 and ln5.

a. ln10 b. $\ln\frac{5}{32}$

Ex 4: Find the exact value of each expression without using a calculator.

a. $\log_7 \sqrt[5]{7}$

b. $\ln e^{12} + \ln e^5$

Ex 5: Expand each logarithmic expression.

a. $\log 3x^2y$

b. $\ln \frac{\sqrt{4x+1}}{8}$

Ex 6: Condense each logarithmic expression.

a. $\frac{1}{3}\log x + 5\log(x-3)$

b. $4\ln(x-4) - 2\ln x$

c. $\frac{1}{5}[\log_3 x + \log_3(x-2)]$